9. Three pistons have the following measurements: $2\frac{5}{16}$ inches, $2\frac{11}{32}$ inches, and $2\frac{9}{64}$ inches. Which piston is the largest?

10. A share of stock sold for $\$23\frac{7}{8}$. The next day it sold for $\$23\frac{3}{4}$. Did the price of the stock go up or down?

UNIT 26. Improper Fractions and Mixed Numbers

WORDS TO KNOW

We have defined a fraction as a part of a unit. However, there are times when a fraction means *more* than a unit. When you buy 6 quarts of milk, for example, you have $\frac{6}{4}$ of a gallon. But $\frac{6}{4}$ of a gallon is the same as $1\frac{1}{2}$ gallons (1 gallon = 4 quarts). We call $\frac{6}{4}$ an **improper fraction.** An improper fraction is a fraction whose numerator is *as large as or larger than* its denominator.

We call $1\frac{1}{2}$ a **mixed number,** and define a mixed number as a number made up of whole units plus a fraction of a unit.

A fraction that is a part of a unit (whose numerator is smaller than its denominator) is called a **proper fraction.**

Since fractions represent parts of whole units, it is said to be "improper" to write a fraction that contains more than one whole unit. Suppose you want to watch a television drama that lasts for 90 minutes, and you want to know what time the show will be over. Before figuring out the problem, you change the time from $\frac{90}{60}$ of an hour to $1\frac{1}{2}$ hours. What you have done is to change an *improper fraction* to a *mixed number.*

A fraction whose numerator is as large as or larger than its denominator is called an **improper fraction.** Examples of improper fractions are $\frac{6}{4}$, $\frac{5}{4}$, and $\frac{4}{4}$.

A number that contains both whole units and a fraction of a unit is called a **mixed number.** Examples of mixed numbers are $3\frac{2}{3}$, $2\frac{1}{4}$, and $1\frac{1}{8}$.

When writing the answer to any calculation, we usually change any improper fractions to mixed numbers or to whole numbers. To change an improper fraction to a mixed number or to a whole number, carry out the indicated division by *dividing the denominator into the numerator.*

A good way to remember which part of the fraction goes inside the division box is to draw the division box next to the bottom number of the fraction and let the top number "fall into" the division box.

EXAMPLE 1. Change $\frac{12}{3}$ to a whole number.

Solution:

Step 1: Draw the division box next to the denominator and let the numerator "fall in."

$$3\overline{)12}$$

Step 2: Do the division.

$$\begin{array}{r} 4 \\ 3\overline{)12} \\ \underline{12} \end{array}$$

Answer: $\frac{12}{3} = 4$

Example 2. Change $\frac{19}{4}$ to a mixed number.

Solution:

$$\begin{array}{r} 4\frac{3}{4} \\ 4\overline{)19} \\ \underline{16} \\ 3 \end{array}$$

Answer: $\frac{19}{4} = 4\frac{3}{4}$

There are times when it is necessary to change a mixed number to an improper fraction.

Example 3. Change $5\frac{3}{8}$ to an improper fraction.

Solution: To do this problem, you are really finding how many eighths are contained in $5\frac{3}{8}$. Set up the problem by rewriting the same denominator:

$$5\frac{3}{8} = \frac{\ }{8}$$

Step 1: To find the new numerator, *multiply the denominator in the fraction part of the mixed number by the whole number.* The denominator of $\frac{3}{8}$ is 8. Therefore, $8 \times 5 = 40$.

$$8 \times 5 = 40 \qquad 5\frac{3}{8} = \frac{\ }{8}$$

Step 2: Now, *add the numerator of the fraction part of the mixed number* to the product you just got: $40 + 3 = 43$. Write this new numerator over the same denominator.

$$8 \times 5 = 40 \qquad 5\frac{3}{8} = \frac{43}{8} \qquad 40 + 3 = 43$$

Answer: $5\frac{3}{8} = \frac{43}{8}$

With practice, you can find the new numerator mentally. In the above problem, you would think: "$8 \times 5 = 40$, and $40 + 3 = 43$."

$$5\frac{3}{8} = \frac{43}{8}$$

Example 4. Change $6\frac{2}{3}$ to an improper fraction.

Solution: Set up the problem:

$$6\frac{2}{3} = \frac{\ }{3}$$

Think: "$3 \times 6 = 18$, and $18 + 2 = 20$." Write the 20 over the same denominator.

$$6\frac{2}{3} = \frac{\ }{3} \qquad 6\frac{2}{3} = \frac{20}{3}$$

Answer: $6\frac{2}{3} = \frac{20}{3}$

Remember

1. To change an improper fraction to a mixed number: Divide the denominator into the numerator.
2. To change a mixed number to an improper fraction: Multiply the denominator by the whole number and add the numerator. This gives you the new numerator, which you write over the same denominator.

EXERCISES

In 1–10, change each improper fraction to a mixed number or to a whole number.

Sample Solutions

a. $\frac{17}{5}$ $\quad 3\frac{2}{5}$; $5\overline{)17}$, 15, $\frac{2}{5}$

b. $\frac{12}{4}$ $\quad 3$; $4\overline{)12}$, 12

1. $\frac{11}{8}$ 2. $\frac{15}{3}$ 3. $\frac{16}{15}$ 4. $\frac{17}{8}$ 5. $\frac{18}{3}$

6. $\frac{23}{6}$ 7. $\frac{17}{7}$ 8. $\frac{23}{8}$ 9. $\frac{32}{6}$ 10. $\frac{15}{4}$

In 11–20, change each mixed number to an improper fraction.

Sample Solutions

a. $3\frac{2}{3} = \frac{11}{3}$
$3 \times 3 = 9$, and $9 + 2 = 11$.

b. $1\frac{5}{8} = \frac{13}{8}$
$8 \times 1 = 8$, and $8 + 5 = 13$.

11. $5\frac{5}{6}$ 12. $12\frac{2}{3}$ 13. $6\frac{3}{4}$ 14. $4\frac{3}{7}$ 15. $15\frac{1}{2}$

16. $7\frac{4}{5}$ 17. $6\frac{5}{16}$ 18. $3\frac{9}{12}$ 19. $8\frac{1}{3}$ 20. $10\frac{3}{4}$

APPLICATION PROBLEMS

Sample Solution

A film version of *War and Peace* runs for 390 minutes. Using the fact that 1 minute = $\frac{1}{60}$ of an hour, change the running time to hours.

$390 \text{ min.} = \frac{390}{60} = \frac{39}{6}$ hr.

$\frac{39}{6} = 6\overline{)39}$ $6\frac{1}{2}$
36
3
$\frac{3}{6}$

$6\frac{1}{2}$ hr.

1. Mr. Wilson gave out 12 quarters to his grandchildren. How much money in dollars did he give out?
(*Hint*: One quarter = $\frac{1}{4}$ of a dollar.)

2. Cathy wants 50 slices of pizza for a club meeting. If a slice is $\frac{1}{8}$ of a pizza, how many pizzas are needed? _____

3. A pail holds 12 quarts of water. What is the capacity in gallons? (*Hint:* 1 quart = $\frac{1}{4}$ gallon.) _____

4. Julio lived in Trenton 42 months. How many years was that? _____

5. A living room rug is 14 feet long by 10 feet wide. How many yards long is the rug? (*Hint:* 1 foot = $\frac{1}{3}$ yard.) _____

   How many yards wide is the rug? _____

SAMPLE SOLUTION

How many $\frac{1}{4}$ cups of flour can you get from a bag that holds $6\frac{3}{4}$ cups?

$6\frac{3}{4} = \frac{27}{4}$

27 $\frac{1}{4}$ cups

6. How many $\frac{1}{8}$-inch-thick slices of cheese can be cut from a piece of cheese that measures $4\frac{3}{8}$ inches? _____

7. How many $\frac{1}{2}$ teaspoons of honey can be poured from a jar that holds $12\frac{1}{2}$ teaspoons? _____

8. Small nails are packed in $\frac{1}{4}$-pound boxes. How many $\frac{1}{4}$-pound boxes can be packed from $15\frac{3}{4}$ pounds of nails? _____

9. How many $\frac{1}{2}$ dozen eggs are there in $6\frac{1}{2}$ dozen eggs? _____

10. A box contains 15 packages of batteries. If each package holds 3 batteries, how many dozen batteries are there in the box? (*Hint:* What fraction of a dozen is 3?) _____

Review of Part VII (Units 22–26)

In 1–12, change each fraction to a higher equivalent that has the given denominator.

1. $\frac{4}{5} = \frac{\quad}{20}$
2. $\frac{3}{7} = \frac{\quad}{28}$
3. $\frac{3}{5} = \frac{\quad}{30}$
4. $\frac{5}{8} = \frac{\quad}{40}$
5. $\frac{5}{6} = \frac{\quad}{42}$
6. $\frac{9}{15} = \frac{\quad}{45}$
7. $\frac{7}{12} = \frac{\quad}{48}$
8. $\frac{11}{16} = \frac{\quad}{48}$