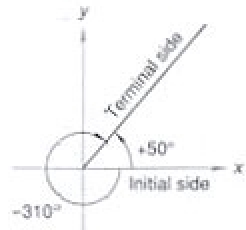
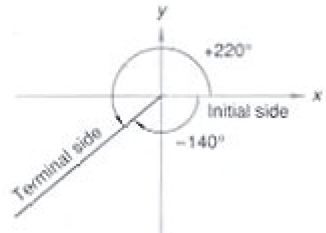
LESSON 36 *Angles Greater Than 360° • Sums of Trigonometric Functions • Boat-in-the-River Problems*

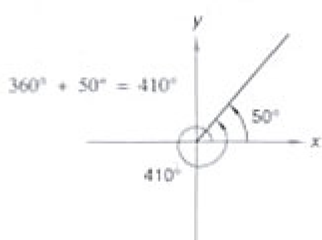
36.A angles greater than 360°

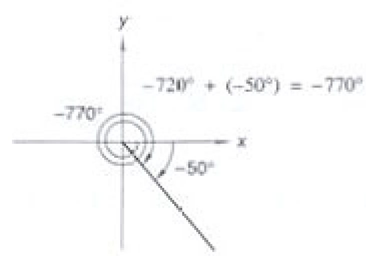
In polar coordinates, we measure positive angles counterclockwise from the positive x axis and measure negative angles clockwise from the positive x axis. Thus, the positive x axis is the initial side of the angle.





In the left-hand figure, we see that +50° and −310° have the same terminal side, and we say that these angles are **coterminal angles.** In the right-hand figure, we see that +220° and −140° are also coterminal angles. For many purposes, we can think of coterminal angles as being the same angle but with a different name. **Angles that differ by integer multiples of 360° are coterminal angles.** Two times around counterclockwise would be 720°, so angles that differ by 720° are coterminal angles. Three times around counterclockwise would be 1080°, so angles that differ by 1080° are also coterminal angles, as are angles that differ by 1440° for four times around counterclockwise, etc.





On the left, we see that 50° and 410° are coterminal angles because 410° is once around for 360° plus 50° more. On the right, we see that −50° and −770° are coterminal angles because −770° is twice around for −720° plus another −50°.

36.B sums of trigonometric functions

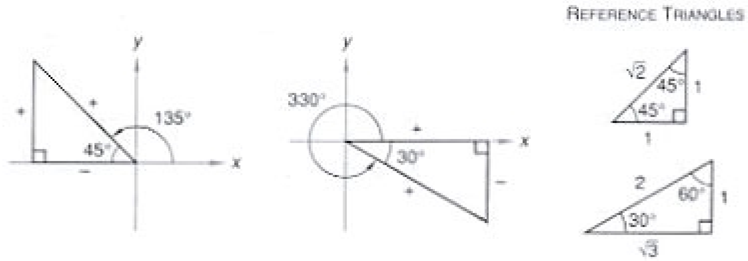
Problems that require the addition of trigonometric functions of angles in all quadrants will provide practice that will lead to greater understanding of the values of trigonometric functions and will also give practice in simplifying expressions that contain radicals. The three expressions shown here are equivalent expressions.

(a) $-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}$ (b) $-\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{3}$ (c) $\frac{-3\sqrt{2} - 2\sqrt{3}}{6}$

Many people prefer expressions that do not have radicals in the denominator. In this case, they would prefer expressions (b) and (c). In this book, we will combine the parts as in (c) because of the practice this procedure provides and not because this form of answer is necessarily more desirable.

example 36.1 Evaluate: $\cos 135° + \tan 330°$

solution It is always helpful to sketch the problem and note the related angles. We also draw the reference triangles.



The cosine of 45° is $1/\sqrt{2}$, and the cosine is negative in the second quadrant.

$$\cos 135° = -\cos 45° = -\frac{1}{\sqrt{2}}$$

The tangent of 30° is $1/\sqrt{3}$, and the tangent is negative in the fourth quadrant.

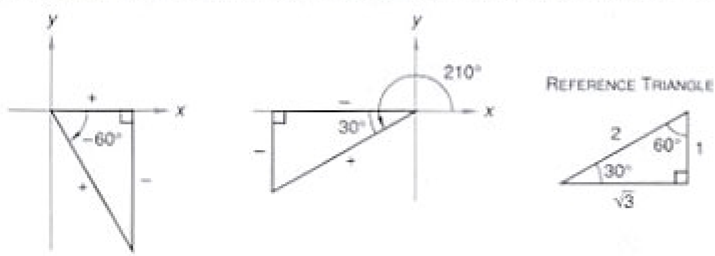
$$\tan 330° = -\tan 30° = -\frac{1}{\sqrt{3}}$$

Therefore, we have

$$\begin{aligned} \cos 135° + \tan 330° &= -\cos 45° - \tan 30° && \text{substituted} \\ &= -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} && \text{sum of functions} \\ &= -\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{3} && \text{rationalized denominators} \\ &= \mathbf{\frac{-3\sqrt{2} - 2\sqrt{3}}{6}} && \text{added} \end{aligned}$$

example 36.2 Evaluate: $\cos(-60°) + \cos 210°$

solution First we sketch the problem and note the related angles. We also draw the reference triangle.



The cosine of 60° is 1/2, and the cosine is positive in the fourth quadrant.

$$\cos(-60°) = \cos 60° = \frac{1}{2}$$

The cosine of 30° is $\sqrt{3}/2$, and the cosine is negative in the third quadrant.

$$\cos 210° = -\cos 30° = -\frac{\sqrt{3}}{2}$$

Therefore, we have

$$\cos(-60°) + \cos 210° = \cos 60° - \cos 30° \quad \text{substituted}$$

$$= \frac{1}{2} - \frac{\sqrt{3}}{2} \quad \text{sum of functions}$$

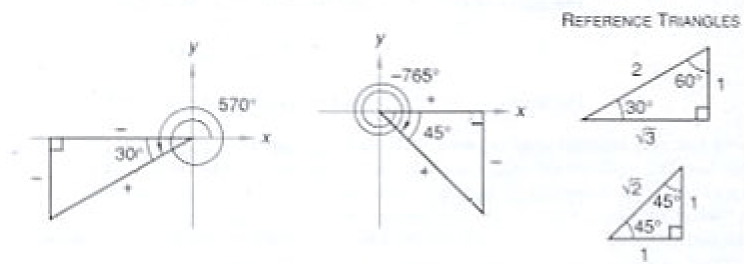
$$= \frac{1 - \sqrt{3}}{2} \quad \text{added}$$

example 36.3 Evaluate: $\cos 570° + \sin(-765°)$

solution We begin by reducing the absolute values of the angles 360° at a time until we get an angle whose measure is less than 360°.

$$570° - 360° = 210° \qquad 765° - 360° - 360° = 45°$$

Now we sketch the angles and note the related angles. We also draw the reference triangles.



The cosine of 30° is $\sqrt{3}/2$, and the cosine is negative in the third quadrant.

$$\cos 570° = \cos 210° = -\cos 30° = -\frac{\sqrt{3}}{2}$$

The sine of 45° is $1/\sqrt{2}$, and the sine is negative in the fourth quadrant.

$$\sin(-765°) = \sin(-45°) = -\sin 45° = -\frac{1}{\sqrt{2}}$$

Therefore, we have

$$\cos 570° + \sin(-765°) = -\cos 30° - \sin 45° \quad \text{substituted}$$

$$= -\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \quad \text{sum of functions}$$

$$= -\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \quad \text{common denominator}$$

$$= \frac{-\sqrt{3} - \sqrt{2}}{2} \quad \text{added}$$

36.C boat-in-the-river problems

We have called problems in which boats travel distances with and against the current **boat-in-the-river problems.** There is a downstream equation in which the rate is the still-water speed of the boat plus the speed of the water $(B + W)$. There is also an upstream equation in which the rate is the still-water speed of the boat minus the speed of the water $(B - W)$.

DOWNSTREAM EQUATION	UPSTREAM EQUATION
(a) $(B + W)T_D = D_D$	(b) $(B - W)T_U = D_U$

The same thought process and equations are applicable in problems in which an airplane flies directly against the wind and directly with the wind, as we see in the next example. The setup of this problem is straightforward, but the solution is a little awkward unless a double-variable substitution is used.

example 36.4 One day Selby found that her plane could fly at 5 times the speed of the wind. She flew 396 miles downwind in $\frac{1}{2}$ hour more than it took her to fly 132 miles upwind. What was the speed of her plane in still air and what was the speed of the wind?

solution First we write the equations. We will use A for the speed of the airplane in still air and W for the speed of the wind. Since distance is in miles and time is in hours, the speeds will be in miles per hour.

Downwind Equation	Upwind Equation
$(A + W)T_D = D_D$	$(A - W)T_U = D_U$

Now we substitute $5W$ for A, 396 and 132 for the distances, and $T_U + \frac{1}{2}$ for T_D.

$$(5W + W)\left(T_U + \frac{1}{2}\right) = 396 \qquad (5W - W)T_U = 132$$

Next we multiply, simplify, and get

$$6WT_U + 3W = 396 \qquad 4WT_U = 132$$

Now, if we have worked a problem like this one before, we note that the right-hand equation can be solved for WT_U, and this value can be used for WT_U in the left-hand equation. First we solve the right-hand equation for WT_U.

$$4WT_U = 132 \qquad \text{equation}$$

$$WT_U = 33 \qquad \text{divided by 4}$$

Now we use 33 in place of WT_U in the left-hand equation and solve.

$$6(33) + 3W = 396 \qquad \text{substituted}$$

$$198 + 3W = 396 \qquad \text{multiplied}$$

$$W = \mathbf{66\ mph} \qquad \text{solved}$$

We were given that A equals $5W$, so

$$A = 5(66)$$

$$A = \mathbf{330\ mph}$$

problem set 36

1. Bruce found that his plane could only fly at 4 times the speed of the wind. He flew 800 miles downwind in 1 hour more than it took to fly 300 miles upwind. What was the speed of the plane in still air and what was the speed of the wind?
2. Twenty percent of the number of whites exceeded the number of reds by 10. Also, three fifths of the number of blues was exactly equal to 3 times the number of reds. How many of each were there if ten percent of the sum of the reds and blues was 94 less than the number of whites?
3. Erica needed to finish with $3\sqrt{2}$, but her answer was $2/\sqrt{6}$. By what number should she have multiplied her number to get the answer she needed?
4. Jan traveled m miles at p miles per hour and arrived 2 hours early. How fast should she have traveled in order to arrive on time?
5. If the length of a rectangle is increased by 20% and the width of the same rectangle is decreased by 20%, what is the percent change in the area?
6. Find the number that is $\frac{2}{7}$ of the way from $-3\frac{1}{7}$ to $4\frac{1}{7}$.