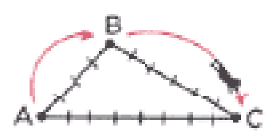
Courtesy of the Calaveras County Jumping Frog Jubilee

Lesson 4

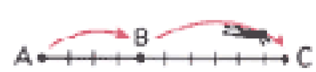
The Triangle Inequality Theorem

The Calaveras Frog Jump is held each spring in Angels Camp, California. Hundreds of frogs compete in trying to jump the farthest.

Suppose that a frog jumps 4 feet and then jumps 6 feet. Is it possible that it could end up 8 feet from its starting point? The figure at the right shows that the answer is yes.

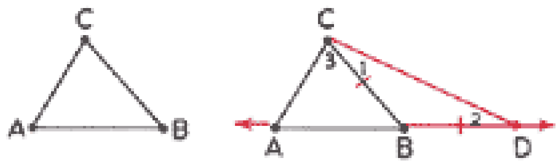


Could the frog jump 4 feet, then jump 6 feet, and end up 12 feet from its starting point? The answer to this question is no. If the frog does not change direction in making its second jump, it will land 10 feet from its starting point. If it does change direction, it will end up *less* than 10 feet from its starting point. This can be proved by means of a useful fact known as the Triangle Inequality Theorem.



▸ **Theorem 20** (The Triangle Inequality Theorem)
The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

The proof of this theorem, like those of the other inequality theorems that we have proved, requires the addition of some extra parts to the figure. This time we will construct an isosceles triangle next to the given triangle and then relate its equal angles to other angles in the figure.



Given: △ABC.
Prove: AB + BC > AC.*

Proof.

Statements	*Reasons*
1. ABC is a triangle.	Given.
2. Draw $\overleftrightarrow{AB}$.	Two points determine a line.
3. Choose point D on $\overrightarrow{AB}$ so that BD = BC.	The Ruler Postulate.
4. Draw $\overline{CD}$.	Two points determine a line.
5. ∠1 = ∠2.	If two sides of a triangle are equal, the angles opposite them are equal.
6. ∠ACD = ∠3 + ∠1.	Betweenness of Rays Theorem.
7. ∠ACD > ∠1.	The "whole greater than its part" property.
8. ∠ACD > ∠2.	Substitution (steps 5 and 7).
9. In △ACD, AD > AC.	If two angles of a triangle are unequal, the sides opposite them are unequal and the longer side is opposite the larger angle.
10. AD = AB + BD.	Betweenness of Points Theorem.
11. AB + BD > AC.	Substitution (steps 9 and 10).
12. AB + BC > AC.	Substitution (steps 3 and 11).

*Also, AC + CB > AB and BA + AC > BC. These inequalities can be proved in the same way.

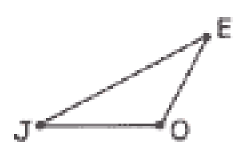
Exercises

Set I

The Triangle Inequality Theorem is true for all triangles.

1. State it as a complete sentence.

Use it to copy and complete the following inequalities for △JOE.



2. JO + OE > ____.
3. JE + ____ > JO.
4. ____ + ____ > OE.

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