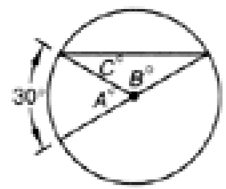
20. $3\sqrt{50} - 2\sqrt{72} + 3\sqrt{162}$

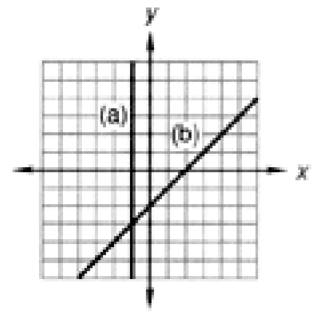
21. $3\sqrt{\frac{3}{7}} + 2\sqrt{\frac{7}{3}}$

22. Find A, B, and C.



23. The altitude of a cylinder is 8 cm. The volume of the cylinder is 32π cm³. What is the radius of the cylinder?

24. Find the equations of lines (a) and (b).



Solve:

25. $\frac{3 - 2x}{4} + \frac{x}{3} = 5$

26. $0.004x - 0.02 = 2.02$

27. Add: $\frac{3}{x} + \frac{2}{x + 2} + \frac{3x}{x^2 + 3x + 2}$

28. Simplify: $\frac{x + 4x}{x}$

29. Expand: $\frac{x^{-2}y}{p}\left(\frac{x^2 p}{y} - \frac{3x^2 y}{p}\right)$

30. Evaluate: $x^2 - xy - x^3$ if $x = \frac{1}{2}$ and $y = \frac{1}{3}$

LESSON 41 Units • Unit multipliers

41.A units

When we attach words to numbers as shown here,

4 ft 6 centimeters 42.78 mph 93 liters

we often call the resulting combinations **denominate numbers**. The Latin prefix for "completely" is *de-*, and the Latin word for "to name" is *nominare*. Thus, *denominate* literally means "completely named." The words are called the **units** of the denominate numbers. Thus, the units in the denominate numbers above are feet, centimeters, miles per hour, and liters.

We find it convenient to use exponential notation to handle units. If we do this, we can handle units the same way we handle numbers or variables. This is especially useful when we multiply or divide units, as we see in these examples.

(a) $\text{ft}^2 \cdot \text{ft} = \mathbf{ft^3}$ (b) $\frac{\text{cm}^3}{\text{cm}} = \mathbf{cm^2}$

(c) $\frac{\text{yd}^3}{\text{yd}} = \mathbf{yd^2}$ (d) $\frac{\text{in.}^3}{\text{in.}^2} = \mathbf{in.}$

41.B unit multipliers

We remember that any nonzero quantity divided by itself has a value of 1.

$$\frac{x^2}{x^2} = \mathbf{1} \qquad \frac{24.123}{24.123} = \mathbf{1} \qquad \frac{6\text{ ft}^2}{6\text{ ft}^2} = \mathbf{1} \qquad \frac{4\text{ in.}}{4\text{ in.}} = 1$$

Furthermore, we remember that the product of any quantity and 1 is the quantity itself.

$$x^2ym(1) = \mathbf{x^2ym} \qquad \left(4\,\frac{\text{ft}}{\text{sec}}\right)(1) = 4\,\mathbf{\frac{ft}{sec}} \qquad (3\text{ in.})(1) = \mathbf{3\text{ in.}}$$

We know that 12 inches equals 1 foot, so if we write either

$$\frac{12 \text{ in.}}{1 \text{ ft}} \qquad \text{or} \qquad \frac{1 \text{ ft}}{12 \text{ in.}}$$

we have written an expression whose value is 1. We can multiply any expression by either of these terms without changing the value of the expression. We call these terms **unit multipliers** for two reasons: One reason is that the expressions contain units, and the second reason is that the expressions have a value of unity (1). Unit multipliers are very helpful when we want to change one set of units to another set of units.

example 41.1 Use a unit multiplier to change 600 inches to feet.

solution We will use one of the unit multipliers above. We choose the one on the left.

$$600 \text{ in.} \times \frac{12 \text{ in.}}{1 \text{ ft}} = \frac{7200 \text{ in.}^2}{\text{ft}}$$

This answer is not incorrect, but it is not what we want. Let's try again and use the other unit multiplier.

$$600 \cancel{\text{in.}} \times \frac{1 \text{ ft}}{12 \cancel{\text{in.}}} = \mathbf{50 \text{ ft}}$$

This time the inches canceled and we found the desired answer.

example 41.2 Use unit multipliers to convert 44 square feet to square inches.

solution We write what was given and use 1 for a denominator.

$$\frac{44 \text{ ft}^2}{1}$$

We note that square feet (ft^2) is in the numerator. Thus, we will use the unit multiplier that has the abbreviation ft in the denominator. We must use two unit multipliers because we are converting from square feet (ft^2) to square inches (in.^2).

$$\frac{44 \cancel{\text{ft}^2}}{1} \times \frac{12 \text{ in.}}{1 \cancel{\text{ft}}} \times \frac{12 \text{ in.}}{1 \cancel{\text{ft}}} = \mathbf{(44)(12)(12) \text{ in.}^2}$$

The multiplication in this example is relatively easy, but some unit conversion problems will result in very complicated multiplications and divisions. We suggest that these answers be left in the form above or that a pocket calculator be used to get the final numerical answer. These problems are designed to teach unit conversions and are not designed for practice in arithmetic.

We have begun our study of unit conversions with problems that can be solved mentally without the use of unit multipliers. It is recommended that the use of unit multipliers not be avoided in these simple problems because the unit conversion problems that we encounter later will be rather involved. The use of unit multipliers will make these involved conversions straightforward, and the experience that we gain by doing simple problems will prove to be valuable.

example 41.3 Use unit multipliers to convert 42 square yards to square inches.

solution We could use the fact that 1 yard equals 36 inches, but instead we will go from square yards (yd^2) to square feet (ft^2) to square inches (in.^2), a procedure that is recommended because shortcuts can lead to errors.

$$42 \cancel{\text{yd}^2} \times \frac{3 \cancel{\text{ft}}}{1 \cancel{\text{yd}}} \times \frac{3 \cancel{\text{ft}}}{1 \cancel{\text{yd}}} \times \frac{12 \text{ in.}}{1 \cancel{\text{ft}}} \times \frac{12 \text{ in.}}{1 \cancel{\text{ft}}} = \mathbf{42(3)(3)(12)(12) \text{ in.}^2}$$

example 41.4 Use unit multipliers to convert 16 cubic miles (mi^3) to cubic inches (in.^3).